Strain Determination of Wood by Coating (V) : Stress in the Coating in Connection with the Failure

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—Stress in the Coating in Connection with the Failure—

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佐々木光*・満久崇麿*：塗膜による木材のひずみ測定（第5報），塗膜の応力と破壊

Introduction

The analysis of stress and strain of materials by brittle coating is based on cracks of the coating. The law of failure of the coating is, therefore, very important. It has always been assumed that brittle coating breaks when the maximum tensile strain exceeds a critical value[1)4)]. But on the coating "Stresscoat" composed of carbon disulfide, resinlike product and plasticizer, DURELLI et al.[2)] demonstrated experimentally that in points where the states of stress approach the conditions of a singular point or pure compression, this maximum tensile strain law does not represent the failure of either condition, and they gave a tentative analytical expression of the law of failure of Stresscoat by the following experimental formula:

$$(\mu^c-\mu^s)\sigma_1{}^s(\sigma_2{}^s)^2+\sigma_1{}^s=1,$$

in which σ_1, σ_2, and μ are the maximum principal stress, minimum principal stress, and POISSON's ratio, respectively. And the index c and s denote the coating and the specimen, respectively.

The above formula can be rewritten on the stresses in the coating as follows, by use of the relations concerning the stresses between the specimen and the coating:

$$(D_1\sigma_1{}^c+D_2\sigma_2{}^c)\{(\mu^c-\mu^s)(D_2\sigma_1{}^c+D_1\sigma_2{}^c)^2+1\}=1,$$
$$D_1=E^s(1-\mu^c\mu^s)/E^c(1-(\mu^s)^2),$$
$$D_2=E^s(\mu^c-\mu^s)/E^c(1-(\mu^s)^2).$$

The failure criteria of Stresscoat are given by the above formula, and depend not only upon the maximum principal stress, but also upon the minimum principal stress of the coating.

It is obvious that, if the elastic constants of the coating and the specimen were the same, the same state of stress would be found on the common layer of the coating and the specimen, but if these constants were quite different, the sign of the stress in the coating may sometimes become the opposite of that in

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the specimen.

Wood specimens are not isotropic in their mechanical properties, for example, three YOUNG's moduli differing by as much as 150 to 1, three shear moduli differing by as much as 20 to 1, six POISSON's rations differing by as much as 40 to 1, and other properties differing by various amounts[5].

The relations between stresses of the specimen and the coating are, therefore, very different in each direction, for example, the maximum tensile stress of the coating in radial stretching of the wood specimen will be larger than that in longitudinal stretching of the same magnitude.

Thus, in stress and strain analysis of wood specimen, it is especially important whether the coating cracks according to the maximum principal strain law or not, and the interpretation of the crack pattern of the coating is very simple in the former case, but much complicated in the latter.

In this paper, (a) the states of stresses of the coating and the based wood specimen are expressed, (b) the relations between them are numerically evaluated for a practical example, and (c) the failure of the coating, "Daira B", used in previous papers[6][9] are discussed with some simple experimental results.

Stress and Strain in Wood and Coating

1. *State of Stress and Strain in the Top Surface of Wood Specimen.*

The discussions are confined to the wood surfaces in which the two natural axes exist, such as cross section, tangential section and radial section. Let these two natural axes be called x and y, so the surface be called $x-y$ plane. On this surface, there are no external loads, so that the stresses perpendicular to this surface are zero, and there exists a state of plane stress.

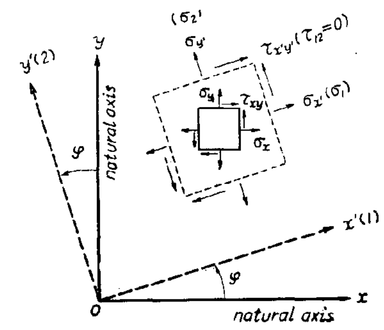

Fig. 1.

If at a point on the plane two mutually perpendicular axes x', y' are chosen in the plane for a voluntary rotation of axes x, y by an angle φ from x towards y (Fig. 1), and normal stresses parallel to these axes and shear stress associated with these axes are denoted by $\sigma_{x'}$, $\sigma_{y'}$, and $\tau_{x'y'}$, the normal stresses parallel to the natural axes σ_x, σ_y and the shear stress associated with these natural axes τ_{xy} are shown as follows[5] :

$$\left.\begin{aligned}\sigma_x&=l_1^2\sigma_{x'}+m_1^2\sigma_{y'}-l_1m_1\tau_{x'y'},\\ \sigma_y&=m_1^2\sigma_{x'}+l_1^2\sigma_{y'}+l_1m_1\tau_{x'y'},\\ \tau_{xy}&=l_1m_1(\sigma_{x'}-\sigma_{y'})+(l_1^2-m_1^2)\tau_{x'y'},\end{aligned}\right\}\quad\cdots\cdots\cdots\cdots\cdots\cdots(1)$$

where

$$l_1=\cos\varphi,\quad m_1=\sin\varphi.$$

If two principal stress axes 1 and 2 are chosen for x' and y' (as in the parentheses in Fig. 1), formulas (1) are reduced to

$$\left.\begin{aligned}\sigma_x&=l_1^2\sigma_1+m_1^2\sigma_2,\\ \sigma_y&=m_1^2\sigma_1+l_1^2\sigma_2,\\ \tau_{xy}&=l_1m_1(\sigma_1-\sigma_2),\end{aligned}\right\}\qquad\cdots\cdots\cdots(2)$$

and in these formulas, φ used in l_1 and m_1, must be

$$\varphi_p=\frac{1}{2}\tan^{-1}\frac{2\tau_{xy}}{\sigma_x-\sigma_y}.\qquad\cdots\cdots\cdots(3)$$

On the other hand the state of strain of wood surface subjected to the plane stress state mentioned above, must properly be in three dimensional. But, if the stress gradient in the wood surface is not so extremely steep, the displacement in the direction normal to the surface has not so large influences on the state of stress and strain in the coatings, and the subsequent discussions can, therefore, be confined to the strain in the surface.

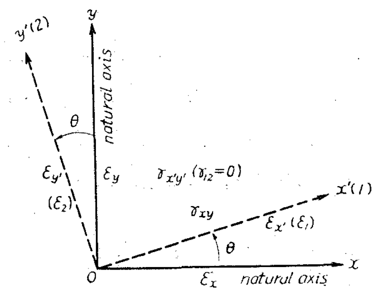

Fig. 2.

In Fig. 2, longitudinal strain parallel to the natural axes of wood ε_x, ε_y and shear strain associated with the axes γ_{xy}, are expressed as follows[5] by those upon the axes x', y' rotated by θ in the plane from x towards y:

$$\left.\begin{aligned}\varepsilon_x&=l_2^2\varepsilon_{x'}+m_2^2\varepsilon_{y'}-l_2m_2\gamma_{x'y'},\\ \varepsilon_y&=m_2^2\varepsilon_{x'}+l_2^2\varepsilon_{y'}+l_2m_2\gamma_{x'y'},\\ \gamma_{xy}&=2l_2m_2(\varepsilon_{x'}-\varepsilon_{y'})+(l_2^2-m_2^2)\gamma_{xy},\end{aligned}\right\}\qquad\cdots\cdots\cdots(4)$$

where, $l_2=\cos\theta$, $m_2=\sin\theta$. And if two principal strain axes 1 and 2 are chosen for x' and y' (as in the parentheses in Fig. 2), formulas (4) are reduced to

$$\left.\begin{aligned}\varepsilon_x&=l_2^2\varepsilon_1+m_2^2\varepsilon_2,\\ \varepsilon_y&=m_2^2\varepsilon_1+l_2^2\varepsilon_2,\\ \gamma_{xy}&=2lm(\varepsilon_1-\varepsilon_2),\end{aligned}\right\}\qquad\cdots\cdots\cdots(5)$$

and in these formulas, θ used in l_2, m_2 must be

$$\theta_p=\frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x-\varepsilon_y}.\qquad\cdots\cdots\cdots(6)$$

Now, according to HOOKE's law, each component of strain in a state of plane stress is given by the expression:

$$\left.\begin{aligned}\varepsilon_x&=\frac{1}{E_x}\sigma_x-\frac{\mu_{yx}}{E_y}\sigma_y,\\ \varepsilon_y&=\frac{1}{E_y}\sigma_y-\frac{\mu_{xy}}{E_x}\sigma_x,\end{aligned}\right\}\qquad\cdots\cdots\cdots(7)$$

$$\gamma_{xy}=\frac{1}{G_{xy}}\tau_{xy}.$$

By substituting these formulas into eq. (6), we obtain

$$\theta_p=\frac{1}{2}\tan^{-1}\frac{2\tau_{xy}}{G_{xy}(2A_x\sigma_x-2A_y\sigma_y)}, \qquad \cdots\cdots(8)$$

where,

$$A_x=(1+\mu_{xy})/E_x, \quad A_y=(1+\mu_{yx})/E_y.$$

Comparison of eqs. (3) and (8) shows that the direction of principal stress and principal strain in wood surface do not, in general, coincide, except when they act along an natural axis of wood ($\tau_{xy}=0$). But in isotropic material, they always coincide because the following relations exist:

$$E_x=E_y=E, \quad \mu_{xy}=\mu_{yx}=\mu,$$

$$G_{xy}=\frac{1}{2A_x}=\frac{1}{2A_y}=\frac{E}{2(1+\mu)}.$$

Eqs. (7) may be solved, so as to express the stress-components in terms of the strain-components, and they are

$$\left.\begin{aligned}\sigma_x&=\frac{E_x}{1-\mu_{xy}\mu_{yx}}(\varepsilon_x+\mu_{yx}\varepsilon_y),\\ \sigma_y&=\frac{E_y}{1-\mu_{xy}\mu_{yx}}(\varepsilon_y+\mu_{xy}\varepsilon_x),\\ \tau_{xy}&=G_{xy}\gamma_{xy}.\end{aligned}\right\} \qquad \cdots\cdots(7')$$

And from eqs. (3), (5) and (7′), a relation between directions of principal stress and principal strain in wood surface is given by

$$\varphi_p=\frac{1}{2}\tan^{-1}\frac{2(\varepsilon_1-\varepsilon_2)\sin 2\theta_p}{\beta_{xy}\{(\alpha_x\cos^2\theta_p-\alpha_y\sin^2\theta_p)\varepsilon_1+(\alpha_x\sin^2\theta_p-\alpha_y\cos^2\theta_p)\varepsilon_2\}}, \qquad \cdots\cdots(9)$$

where,

$$\beta_{xy}=\frac{1}{G_{xy}},$$

$$\alpha_x=\frac{1-\mu_{yx}}{1-\mu_{xy}\mu_{yx}}E_x, \quad \alpha_y=\frac{1-\mu_{xy}}{1-\mu_{xy}\mu_{yx}}E_y.$$

This is an important equation in brittle lacquer technique to determine the direction of principal stress which must be calculated from both the direction and the magnitude of principal strains.

Well, let us express principal strains in the top surface of wood in terms of the principal stresses for the convenience of the discussion in the subsequent articles.

From eqs. (5) and (6), principal strains are

$$\left.\begin{matrix}\varepsilon_1\\ \varepsilon_2\end{matrix}\right\}=\frac{1}{2}[(\varepsilon_x+\varepsilon_y)\pm\{(\varepsilon_x-\varepsilon_y)^2+\gamma_{xy}^2\}^{1/2}],$$

and by substituting eqs. (7) and (2) in these equations, the principal strains in wood surface are given in terms of the magnitudes and directions of the principal stresses as follows:

$$\left.\begin{matrix}\varepsilon_1\\ \varepsilon_2\end{matrix}\right\} = \frac{1}{2}[(l_1^2B_x+m_1^2B_y)\sigma_1+(m_1^2B_x+l_1^2B_y)\sigma_2 \pm [\{(l_1^2A_x-m_1^2A_y)\sigma_1+(m_1^2A_x-l_1^2A_y)\sigma_2\}^2+\beta_{xy}^2l_1^2m_1^2(\sigma_1-\sigma_2)^2]^{1/2}], \quad \cdots\cdots(10)$$

where,

$$B_x=(1-\mu_{xy})/E_x,$$
$$B_y=(1-\mu_{yx})/E_y.$$

2. State of Stress and Strain in the Coating.

Since the coating is thin, the stresses perpendicular to the coating throughout the depth of coating are smaller compared with stresses in other directions. Without appreciable error, we can neglect these stresses and assume a state of plane stress to exist in the coating[3)].

The relations between the stress and strain in the coating are

$$\left.\begin{aligned}\varepsilon_1^c&=\frac{1}{E^c}(\sigma_1^c-\mu^c\sigma_2^c),\\ \varepsilon_2^c&=\frac{1}{E^c}(\sigma_2^c-\mu^c\sigma_1^c),\end{aligned}\right\} \quad \cdots\cdots\cdots\cdots\cdots\cdots(11)$$

where index c is used to identify the stresses, strains and elastic constants of coating from those of the base specimen.

If the adhesion between the coating and the base specimen are assumed to be perfect, the strain in the coating will be then identical to the strain in the top surface of the specimen:

$$\varepsilon_1^c=\varepsilon_1, \quad \varepsilon_2^c=\varepsilon_2. \quad \cdots\cdots\cdots\cdots\cdots\cdots(12)$$

Although the principal directions of stresses and strains of the coating are the same and they will also coincide with those of strains in the top surface of the wood specimen, they do not, in general, coincide with the principal directions of stresses in the top surface of the specimen, as mentioned previously. In accordance with eqs. (12), we can combine the eqs. (10) and (11).

$$\sigma_1^c-\mu^c\sigma_2^c=\frac{E^c}{2}[(l_1^2B_x+m_1^2B_y)\sigma_1+(m_1^2B_x+l_1^2B_y)\sigma_2 + [\{(l_1^2A_x-m_1^2A_y)\sigma_1+(m_1^2A_x-l_1^2A_y)\sigma_2\}^2+\beta_{xy}^2l_1^2m_1^2(\sigma_1-\sigma_2)^2]^{1/2}],$$

$$\sigma_2^c-\mu^c\sigma_1^c=\frac{E^c}{2}[(l_1^2B_x+m_1^2B_y)\sigma_1+(m_1^2B_x+l_1^2B_y)\sigma_2 - [\{(l_1^2A_x-m_1^2A_y)\sigma_1+(m_1^2A_x-l_1^2A_y)\sigma_2\}^2+\beta_{xy}^2l_1^2m_1^2(\sigma_1-\sigma_2)^2]^{1/2}].$$

By solving these equations, we obtain

$$\left.\begin{matrix}\sigma_1^c\\ \sigma_2^c\end{matrix}\right\} = C_1\{(l_1^2B_x+m_1^2B_y)\sigma_1+(m_1^2B_x+l_1^2B_y)\sigma_2\}$$

$$\pm C_2[\{(l_1^2A_x-m_1^2A_y)\sigma_1+(m_1^2A_x-l_1^2A_y)\sigma_2\}^2+\beta_{xy}^2l_1^2m_1^2(\sigma_1-\sigma_2)^2]^{1/2}, \quad\cdots(13)$$

where,

$$C_1=\frac{E^c}{2(1-\mu^c)},\quad C_2=\frac{E^c}{2(1+\mu^c)}$$

These are the equations which express the relations between the principal stresses of the coating and those of the wood specimens. For the convenience of plotting these equations, dividing both members in eqs. (13) by σ_1, these formulas become:

$$\left.\begin{matrix}\sigma_1^c/\sigma_1\\ \sigma_2^c/\sigma_1\end{matrix}\right\} = C_1\left\{(l_1^2B_x+m_1^2B_y)+(m_1^2B_x+l_1^2B_y)\frac{\sigma_2}{\sigma_1}\right\}\pm C_2\Big[\{(l_1^2A_x-m_1^2A_y)^2$$

$$+\beta_{xy}^2l_1^2m_1^2\}+2\{(l_1^2A_x-m_1^2A_y)(m_1^2A_x-l_1^2A_y)-\beta_{xy}^2l_1^2m_1^2\}\frac{\sigma_2}{\sigma_1}$$

$$+\{(m_1^2A_x-l_1^2A_y)^2+\beta_{xy}^2l_1^2m_1^2\}\left(\frac{\sigma_2}{\sigma_1}\right)^2\Big]^{1/2}. \quad\cdots\cdots(14)$$

3. Numerical Discussion in a Practical Example.

For the most simple and practical example, we discuss the case in which the principal axes of stress and strain in wood surface are coincide with the natural axes. The discussions will be applied also to the problems of thermal stresses or of residual stresses by changing moisture content of wood.

If the principal axis 1 coincide with the x axis ($\varphi_p=0$), subscripts 1 and 2 can be expressed by x and y respectively in eqs. (14), and they are reduced to

$$\left.\begin{matrix}\sigma_x^c/\sigma_x\\ \sigma_y^c/\sigma_x\end{matrix}\right\} = (C_1B_x\pm C_2A_x)+(C_1B_y\mp C_2A_y)\frac{\sigma_y}{\sigma_x}.$$

Writing again these equations by using elastic constants, they are

$$\left.\begin{matrix}\sigma_x^c/\sigma_x\\ \sigma_y^c/\sigma_x\end{matrix}\right\} = \left(\frac{E^c}{2(1-\mu^c)}\frac{1-\mu_{xy}}{E_x}\pm\frac{E^c}{2(1+\mu^c)}\frac{1+\mu_{xy}}{E_x}\right)+\left(\frac{E^c}{2(1-\mu^c)}\frac{1-\mu_{yx}}{E_y}\mp\frac{E^c}{2(1+\mu^c)}\frac{1+\mu_{yx}}{E_y}\right)\frac{\sigma_y}{\sigma_x},$$

or,

$$\left.\begin{aligned}\frac{\sigma_x^c}{\sigma_x}\frac{E_x}{E^c}&=\frac{1-\mu^c\mu_{xy}}{1-(\mu^c)^2}+\frac{\mu^c-\mu_{yx}}{1-(\mu^c)^2}\frac{E_x}{E_y}\frac{\sigma_y}{\sigma_x},\\ \frac{\sigma_y^c}{\sigma_x}\frac{E_x}{E^c}&=\frac{\mu^c-\mu_{xy}}{1-(\mu^c)^2}+\frac{1-\mu^c\mu_{yx}}{1-(\mu^c)^2}\frac{E_x}{E_y}\frac{\sigma_y}{\sigma_x}.\end{aligned}\right\} \quad\cdots\cdots(15)$$

If the principal axis 1 coincide with the natural axis $y(\varphi_p=\pi/2)$, we can conduct the equations of similar form to the above:

$$\left.\begin{aligned}\frac{\sigma_y^c}{\sigma_y}\frac{E_y}{E^c}&=\frac{1-\mu^c\mu_{yx}}{1-(\mu^c)^2}+\frac{\mu^c-\mu_{xy}}{1-(\mu^c)^2}\frac{E_y}{E_x}\frac{\sigma_x}{\sigma_y},\\ \frac{\sigma_x^c}{\sigma_y}\frac{E_y}{E^c}&=\frac{\mu^c-\mu_{yx}}{1-(\mu^c)^2}+\frac{1-\mu^c\mu_{xy}}{1-(\mu^c)^2}\frac{E_y}{E_x}\frac{\sigma_x}{\sigma_y}.\end{aligned}\right\} \quad\cdots\cdots(16)$$

Now, let us assume the natural axes x and y to be the longitudinal and radial axes of a tree, respectively, and cite Lawson cypress (*Chamaecyparis lawsoniana* PARL.) as an example. The elastic constants of Lawson cypress are[6)]

$$E_x = 14.6 \times 10^4 \text{ kg/cm}^2, \quad E_y = 0.85 \times 10^4 \text{ kg/cm}^2,$$

$$\mu_{xy} = 0.36, \quad \text{and} \quad \mu_{yx} = 0.021.$$

And if the POISSON's ratio of the coating μ^c is equal to 0.3, eqs. (15) become

$$\left.\begin{aligned} \frac{\sigma_x^c}{\sigma_x}\frac{E_x}{E^c} &= 0.98 + 5.3\frac{\sigma_y}{\sigma_x}, \\ \frac{\sigma_y^c}{\sigma_x}\frac{E_x}{E^c} &= -0.066 + 19\frac{\sigma_y}{\sigma_x}. \end{aligned}\right\} \qquad \text{(15a)}$$

If $\mu^c = 0.4$, they are

$$\left.\begin{aligned} \frac{\sigma_x^c}{\sigma_x}\frac{E_x}{E^c} &= 1.02 + 7.7\frac{\sigma_y}{\sigma_x}, \\ \frac{\sigma_y^c}{\sigma_x}\frac{E_x}{E^c} &= 0.048 + 20\frac{\sigma_y}{\sigma_x}. \end{aligned}\right\} \qquad \text{(15b)}$$

If $\mu^c = 0.5$, they are

$$\left.\begin{aligned} \frac{\sigma_x^c}{\sigma_x}\frac{E_x}{E^c} &= 1.08 + 11\frac{\sigma_y}{\sigma_x}, \\ \frac{\sigma_y^c}{\sigma_x}\frac{E_x}{E^c} &= 0.18 + 23\frac{\sigma_y}{\sigma_x}, \end{aligned}\right\} \qquad \text{(15c)}$$

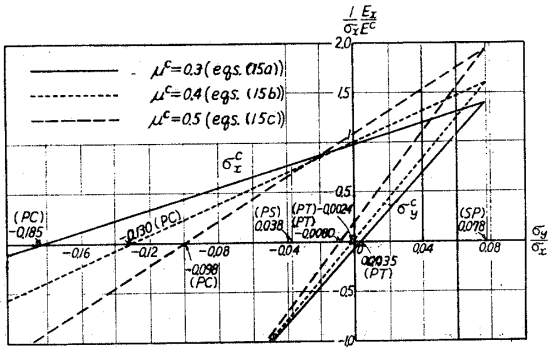

Fig. 3. Stresses in the coating as functions of the state of stress in the specimen of Lawson cypress (eqs. (15)).

These equations are plotted in Fig. 3.

When the wood specimen is subjected to the uniaxial load parallel to the grain ($\sigma_y = 0$), the sign of σ_x^c and σ_y^c is the same as σ_x, if μ^c is larger than μ_{xy}, and this means that for pure compression in the specimen both principal stresses in the coating are compressive, although one of the strains is tensile. The value of μ^c depends on the composition and the drying condition of the coating, and the only one information for Stresscoat by STOKEY[10)] indicates the values of 0.42~0.45.

The values of the principal stresses in the coating can be evaluated if the YOUNG's modulus of the coating E^c is known. The value of E^c depends also on the composition and the drying condition of the coating. NISHIHARA et al.[8)] observed that the YOUNG's modulus of the coating Daira B increased from 2.5×10^4 kg/cm^2 to 6×10^4 kg/cm^2 when drying time was prolonged from 4 hr to 10 hr at 60°C. The YOUNG's modulus may be lower than the order of 10^4 kg/cm^2 if the coating is dried only at room temperature, and this is very advantageous for the stress analysis of materials having low YOUNG's modulus such as wood.

In Fig. 3, in order to produce pure tension (PT) in the coating, it is necessary to introduce in the specimen transversal tension equal to 35/10000 the longitudinal tension (if $\mu^c=0.3$) or transversal compression equal to 24/10000 ($\mu^c=0.4$) or 80/10000 ($\mu^c=0.5$) the longitudinal tension. And when the compression σ_y is equal to 185/1000 ($\mu^c=0.3$), 130/1000 ($\mu^c=0.4$) or 98/1000 ($\mu^c=0.5$) the tension σ_y, the stress in the coating $\sigma_x{}^c$ becomes zero (pure compression (PC)). Two equal principal stresses in the coating are produced when σ_y is equal to 78/1000 σ_x (singular point (SP)), and their values are about 78% ($\mu^c=0.5$), 60% ($\mu^c=0.4$), and 44% ($\mu^c=0.3$) higher than the stress produced by only one principal stress σ_y. When the compression σ_y is equal to 38/1000 the tension σ_x (PS), the coating is in the state of pure shear stress.

On the other hand, substitution of the elastic constants of Lawson cypress into eqs. (16) produces the following equations:

If $\mu^c=0.3$,

$$\left.\begin{aligned}\frac{\sigma_y{}^c}{\sigma_y}\frac{E_y}{E^c}&=1.09-0.0038\frac{\sigma_x}{\sigma_y},\\ \frac{\sigma_x{}^c}{\sigma_y}\frac{E_y}{E^c}&=0.31+0.057\frac{\sigma_x}{\sigma_y}.\end{aligned}\right\}\qquad\cdots\cdots\cdots\cdots\cdots\cdots(16a)$$

If $\mu^c=0.4$,

$$\left.\begin{aligned}\frac{\sigma_y{}^c}{\sigma_y}\frac{E_y}{E^c}&=1.18+0.0028\frac{\sigma_x}{\sigma_y},\\ \frac{\sigma_x{}^c}{\sigma_y}\frac{E_y}{E^c}&=0.45+0.059\frac{\sigma_x}{\sigma_y}.\end{aligned}\right\}\qquad\cdots\cdots\cdots\cdots\cdots\cdots(16b)$$

If $\mu^c=0.5$,

$$\left.\begin{aligned}\frac{\sigma_y{}^c}{\sigma_y}\frac{E_y}{E^c}&=1.32+0.0105\frac{\sigma_x}{\sigma_y},\\ \frac{\sigma_x{}^c}{\sigma_y}\frac{E_y}{E^c}&=0.64+0.063\frac{\sigma_x}{\sigma_y}.\end{aligned}\right\}\qquad\cdots\cdots\cdots\cdots\cdots\cdots(16c)$$

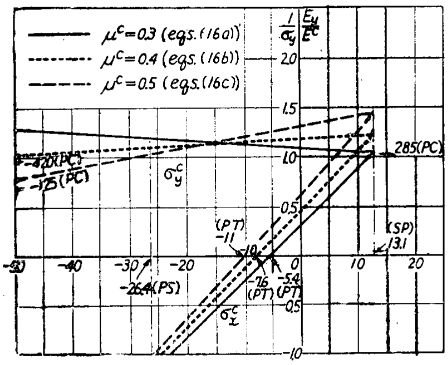

Fig. 4. Stresses in the coating as functions of the state of stress in the specimen of Lawson cypress (eqs. (16)).

These equations are plotted in Fig. 4.

Observation of this figure makes some important facts obviously. The stretching of the specimen in only y direction ($\sigma_x=0$) produces always tensile stresses in both x and y direction of the coating. Pure tension in the coating (PT) is produced when compression σ_x is 5.4 times (if $\mu^c=0.3$), 7.6 times ($\mu^c=0.4$), and 11 times ($\mu^c=0.5$) as large as tension σ_y. If $\mu^c=0.3$, to produce pure compression in the coating (PC), it is necessary to introduce in the specimen tension σ_x equal to 285 times as large as tension σ_y, but if $\mu^c=0.4$ and 0.5, it is necessary compression 240 times and 125 times as large as tension σ_y, respectively. Two equal principal stresses in the coating (SP) are produced when σ_x is 13.1 times as large as σ_y, and their values are about 10.7% (if $\mu^c=0.5$) and 4.2% ($\mu^c=0.4$) higher than the stresses produced by only one principal stress σ_y, but about 4.6% lower when $\mu^c=0.3$. When the compression σ_x is 26.4 times as large as the tension σ_y (PS), the coating is in the state of pure shear.

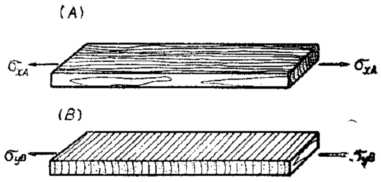

Fig. 5.

As a more simple and concrete example for the discussions above, let us assume two specimens as shown in Fig. 5 being stretched in the same magnitude of strain, and make comparison between the stresses in the coating parallel to the stretching (longitudinal) of these two specimens.

The longitudinal stress in the coating of specimen A is given by substituting $\sigma_y=0$ in the first equation of eqs. (15) :

$$\sigma_{xA}{}^c=\frac{1-\mu^c\mu_{xy}}{1-(\mu^c)^2}E^c\frac{\sigma_{xA}}{E_x}. \qquad \cdots\cdots\cdots\cdots(17)$$

And the longitudinal stress in the coating of specimen B given by substituting $\sigma_x=0$ in the first equation of eqs. (16) :

$$\sigma_{yB}{}^c=\frac{1-\mu^c\mu_{yx}}{1-(\mu^c)^2}E^c\frac{\sigma_{yB}}{E_y}. \qquad \cdots\cdots\cdots\cdots(18)$$

In eqs. (17) and (18), σ_{xA}/E_x and σ_{yB}/E_y are the longitudinal strains of specimen A and B respectively, and they are assumed to be the same :

$$\frac{\sigma_{xA}}{E_x}=\frac{\sigma_{yB}}{E_y}.$$

Consequently, from eqs. (17) and (18) we can obtain the ratio of these stresses :

$$\frac{\sigma_{yB}{}^c}{\sigma_{xA}{}^c}=\frac{1-\mu^c\mu_{yx}}{1-\mu^c\mu_{xy}}. \qquad \cdots\cdots\cdots\cdots(19)$$

Now, let Lawson cypress be cited again as an example, and we can obtain the numerical solution for some possible values of μ^c :

$$\text{for } \mu^c=0.3,\quad \frac{\sigma_{yB}{}^c}{\sigma_{xA}{}^c}=1.12,$$

$$\text{for } \mu^c=0.4,\quad \frac{\sigma_{yB}{}^c}{\sigma_{xA}{}^c}=1.15,$$

$$\text{for } \mu^c=0.5,\quad \frac{\sigma_{yB}{}^c}{\sigma_{xA}{}^c}=1.22,$$

From this, it is obvious that the maximum principal stress in the coating on the wood specimen stretched in the radial direction is 12~22% bigger than that on the wood specimen stretched in the fiber direction in the same magnitude of strain.

Failure of Brittle Coating "Daira *B*"

NISHIHARA et al.[8] observed crack patterns of a coating on an aluminium circular plate with clamped edges loaded perpendicular to the plate with a uniformly distributed pressure. And contrary to the conclusion lead by DURELLI et al.* on Stresscoat, they concluded that a brittle coating, Daira *B*, composed of phenol resin and titanic white dissolved in a mixture of benzene, toluene, and xylene, cracked according to the maximum principal strain law when dried at temperature of 60° to 70°C. And this may be applied effectively to the analysis of stress and strain in wood specimens.

But, when wood specimens are exposed in high temperature, they will be dried and shrunk, and thus the state of stress and strain in the coating will be unstable. And there are considerable differences of thermal expansion between the coating and the wood specimen, and especially it is isotropic in the former but anisotropic in the latter. The state of stress and strain in the coating will, also, be unstable. For these reasons, it is to be desired that the coating is dried at room temperature.

In our previous papers[6)9)], Daira *B* was used and dried at temperature of 10° to 30°C, and the stress and strain analysis of wood specimens were done on the assumption in which the coating cracks according to the maximum principal strain law. But, it is doubtful whether the maximum principal strain law will be followed in this case as well as in the case of Daira *B* dried at higher temperature.

The law of failure of Daire *B* dried at room temperature have not been found. In this study, the same method as NISHIHARA's[8] was taken, but the experiment came to naught because the strain sensitivity** of this coating was not high enough to produce many cracks necessary to conclude the law of failure of the coating within the proportional limit of aluminium plate.

The second experiment was carried out to observe whether the principal strain law is applied on the failure of the coating when the base wood specimen was subjected to the pure compressive load parallel to grain.

* Refer to the introduction of this paper.
** This is defined as the minimum strain necessary to crack the coating in a unidimensional state of stress (sensitivity is defined as the inverse of strain sensitivity).

The test pieces used were stocks of 60 mm (in the fiber direction) by 18 mm (in the tangential direction) by 18 mm (in the radial direction) of Lawson cypress. Thirty specimens were dried in an oven and ten of them were provided for the determination of the POISSON's ratio in the tangential section μ_{LT}, and the other twenty specimens were for the coating test. POISSON's ratios were measured by wire resistance strain gage rosettes and the average value and the standard deviation obtained on the twenty surfaces of the ten specimens was:

$$\mu_{LT}=0.42\pm0.11. \qquad \cdots\cdots\cdots\cdots(20)$$

All of the twenty specimens were coated on each of their two tangential sections with Daira *B* by the same procedure as previous papers[6)9)]. And then, they were dried at temperature of 6°~15°C for 8 days in dried air.

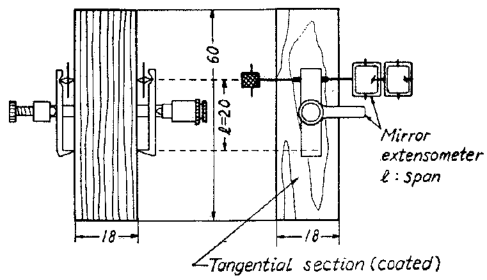

Fig. 6. Specimen for pure compression test parallel to grain.

The compression test was carried out on these coated specimens measuring the longitudinal strain with a mirror extensometer as shown in Fig. 6. The loaded ends of the specimen were greased so as to prevent the friction between the specimen and the crossheads of the testing machine.

When the load was applied, the specimen was extended laterally by $-\mu_{LT}\varepsilon_x$ (ε_x is the longitudinal compressive strain). No cracks of the coating were observed during the load was small, but the longitudinal cracks took place by the load more than a certain value (Photo. 1). With increasing of the load, the crack density increased.

The cracks near the ends of the specimen was rather uniform but the dis-

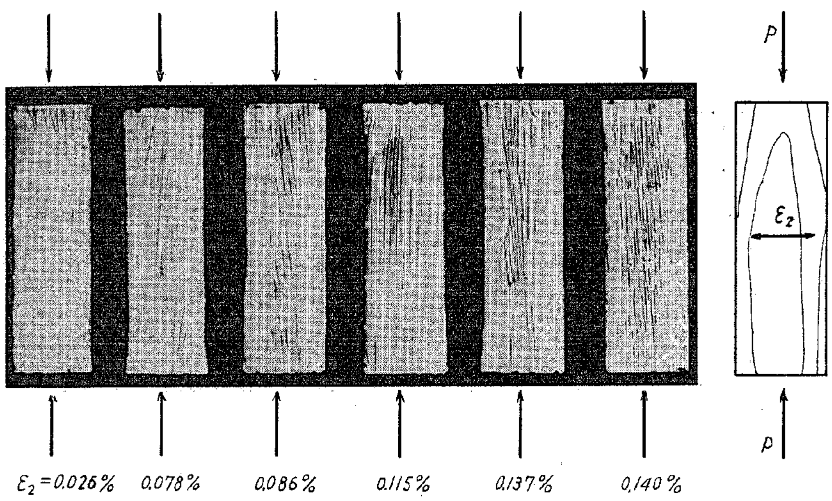

Photo. 1. Cracks of the coating at various stages of the lateral strain of specimens.

tribution of the cracks was not uniform throughout the width of the specimen, and this phenomenon can not be explained only from the contact problems between the specimen and the crosshead.

The lateral strain was calculated from the longitudinal strain observed on each side of the specimen using the value of the POISSON's ratio (expression (20)). Fig. 7 represents the number of the cracks included in the width of the specimen which was plotted as a function of the lateral strain of the specimen. From this figure, it can be concluded that in longitudinal compression of specimen the minimum value of the average lateral strain throughout the width of the specimen necessary to crack the coating is

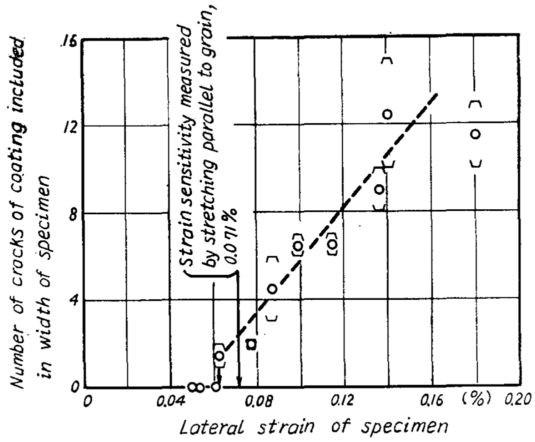

Fig. 7. Cracks of coating parallel to grain as a function of lateral expansion of specimens subjected uniaxial compressive load parallel to grain.

$$\varepsilon_{l0} = 0.062(\%). \qquad \cdots\cdots\cdots\cdots (21)$$

Simultaneously with the above experiment, five strips of 230 mm (in the fiber direction) by 30 mm (in the radial direction) by 6 mm (in the tangential direction) were also coated with Daira B and dried in the same conditions as mentioned above. These strips were available to determine the strain sensitivity of the coating. They were tested as cantilever beams. When one of the beams was loaded, the cracks of the coating on the tension side surface of the beam were produced perpendicularly to the longitudinal axis of the beam. The location of the first crack was measured and the strain sensitivity was determined from the following formula :

$$\varepsilon_0 = \frac{Wh}{2EI} x_0.$$

Where ε_0 is the strain sensitivity of the coating, E is the YOUNG's modulus of the beam, W is the load applied on one end of the beam, I is the moment of inertia of the cross section of the beam with reference to the neutral axis of the beam, h is the height of the beam, and x_0 is the distance between the loaded point and the first crack. The average strain sensitivity and the standard deviation of the coating measured by these five strips were :

$$\varepsilon_0 = 0.071 \pm 0.006(\%) \qquad \cdots\cdots\cdots\cdots (22)$$

The lateral stress in the coating is equal to zero when $\mu^c = \mu_{LT}$, and compressive when $\mu^c > \mu_{LT}$, but tensile when $\mu^c < \mu_{LT}$. In eqs. (15), substituting $\mu_{xy} = \mu_{LT} = 0.42$ and $\sigma_y = 0$, and assuming $\mu^c = 0.3$ for an extreme example, they are

$$\left.\begin{aligned}\frac{\sigma_x{}^c}{E^c}&=0.960\frac{\sigma_x}{E_x},\\ \frac{\sigma_y{}^c}{E^c}&=0.132\frac{\sigma_x}{E_x}.\end{aligned}\right\}\qquad\cdots\cdots\cdots\cdots(23)$$

Replacing σ_x/E_x in the first equation by ε_0 (eq. (22)) and in the second equation by ε_{l0} (eq. (21)), they are

$$\frac{\sigma_x{}^c}{E^c}=0.068,$$

$$\frac{\sigma_y{}^c}{E^c}=0.0082.$$

From these equations, we obtain

$$\frac{\sigma_y{}^c}{\sigma_x{}^c}\doteqdot 0.12.$$

This is a very important fact that even in the extreme case the stress in the first crack produced by the longitudinal pure compressive stress is only 12% of that produced by the longitudinal pure tensile stress. This means that the failure of the coating Daira *B* is possibly independent of the state of stress because the 12% mentioned above can not crack the coating.

While, it is a question that ε_{l0} is smaller than ε_0. This may be explained from the fact that the lateral strain measured by rosette gage corresponds to almost the average value throughout the width of the specimen because the gage length was 16 mm, and therefore the strain at the center portion where the first crack takes place, will be somewhat larger than the value calculated from the POISSON's ratio and the longitudinal strain.

Thus, it is possible that ε_{l0} is almost equal to ε_0, and practically sufficient results are expected by assuming that the coating, Daira *B*, cracks according to the maximum principal strain law also when it is dried at room temperature.

摘　　要

この報告は木材の応力状態とその上に密着する等方性皮膜の応力状態との間に存在する関係について論じたものである。この報告の目的は脆性塗膜で木材の応力を解析する際の基本的な事項を明らかにすることであるが，ここに示す結果は一般の塗装された木材やプラスチック・オーバーレイ合板などの熱応力あるいは水分変化により生ずる内部応力の解析に応用できる。結果を要約すると次の通りである：

(1)　一般に木材の主応力軸は皮膜の主応力軸に一致しない。

(2)　木材の主応力と皮膜の主応力の間には eqs. (13) または (14) の関係が存在する。

(3)　上の関係の数値例を Lawson cypress について示せば，Fig. 3 および Fig. 4 のようになる。

(4)　上述の応力関係を用いて塗膜の破壊についての実験結果を考察すれば，市販の脆性塗

料ダイラーBの破壊は低温で乾燥した場合，高温乾燥の場合と同様，応力に無関係で，最大主ひずみ説に従うと考えられ，木材の応力解析に有利である。

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